CALCULATING PARTICLE VELOCITIES AND CONCENTRATIONS IN THE SPACE ABOVE A PSEUDOFLUIDIZED BED

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We offer a statistical model to calculate the dynamic characteristics of the disperse phase in the space above the bed. We have derived expressions which determine the distribution functions for particles on the basis of velocities and height in the space above the bed, the average values for velocity, flow, and particle concentration as functions of the height to which these particles are ejected at a given initial-velocity distribution. It is demonstrated that the concentration, flow, and mean velocity of the particles at any given height in the space above the bed depends only slightly on the form of the initial distribution, provided that its dispersion is maintained.

The space above the bed can be utilized to carry out technological processes on a par with the core of the bed. To calculate the characteristics of transport within the zone of dynamic ejecta, it is essential that we know the distribution of particles by velocity, concentration, and the flow at any height over the surface of the pseudofluidized bed. An exponential relationship between particle concentration in the space above the bed and the ascending height [1-5] is used extensively in the literature. Such a relationship has been validated experimentally [1] or by proceeding from the scattering of the initial particle velocity [2]. No relationships are presented here for the calculation of the velocity and flow of particles at various heights in the space above the bed. A statistical approach is utilized in [6-8] to describe the zone of dynamic ejecta. Proceeding from the distribution by particle velocities at the boundary between the core of the bed and the space above the bed, we determine the particle concentration at a given height within the confines of the zone [6, 7] or we estimate the intensity of the interphase exchange of heat [8].

The purpose of the present study is to determine, within the framework of a statistical approach analogous to [6-8], the characteristics of the disperse phase in the space above the bed, i.e., the average velocity, the flow and the concentration, as well as the corresponding distribution functions in the zone of dynamic ejecta, proceeding from the given distribution of particles by expulsion velocities at the boundary between the bed and the space above the bed.

Ejection of particles out of the bed is directly related to the destruction of bubbles at the bed surface, accompanied by a redistribution of the gas flow [9-11]. Although this process is randomly localized with respect to time and space in the case of a solitary bubble, given sufficiently prolonged periods of observation it may most probably be regarded as a steady-state process characterized by some average concentration C_0 for the particles being expelled out of the bed, with an average expulsion velocity $\overline{W}_0$, and a expulsion-velocity distribution function $\varphi(w_0)$. As the particles rise up, their average velocity changes and the original distribution function becomes transformed. In this case, owing to the differences in the original velocities simultaneously departing particles diverge, their initial concentration diminishes as the particles rise, and a distribution function with respect to height is established. Starting from some level above the surface of the bed (the maximum height of particle expulsion with minimum initial velocity), we note an additional reduction in the concentration of the disperse material as a consequence of the reduction in the number of ascending particles. The totality of particles distributed through the expulsion height and, at the instant at which they attain this height, exhibiting zero velocity, form the original distribution for the descending flow of disperse material.

Thus, under the steady-state conditions in one-dimensional approximation the space above the bed (the zone of dynamic ejecta) can be represented as a vertical channel in the gravitational field, filled with particles moving in opposite directions. The ascending flow is comprised of particles for which, at the lower boundary (the surface of the pseudofluidized bed) the distribution over expulsion velocities is specified. The descending flow is comprised of particles for which the distribution over expulsion height is specified in the upper portion of the space above the bed. Although each of these mutually penetrating flows vary within the limits of the dynamic-ejecta zone, under steady-state conditions their algebraic sum, i.e., the resulting flow, remains unchanged and equal to zero both at any point in the space above the bed and outside

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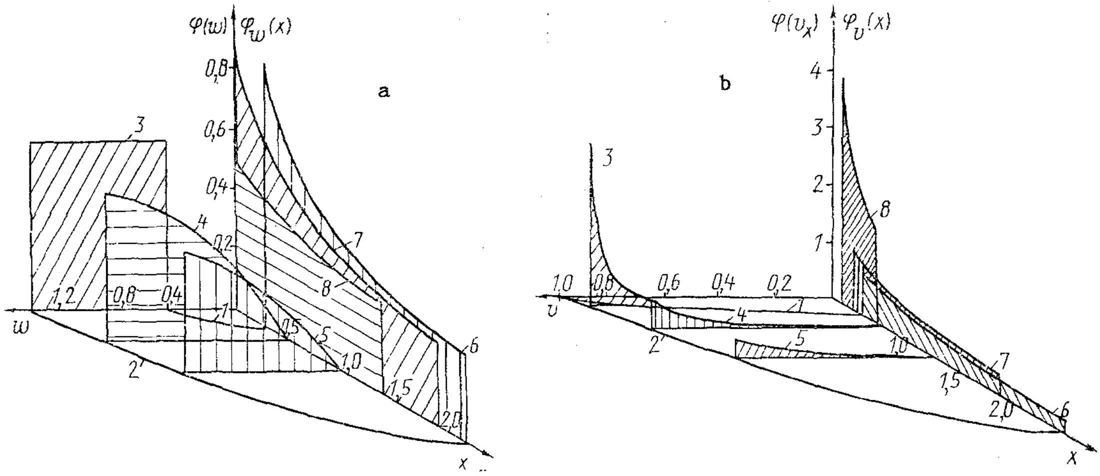

Fig. 1. Densities of particle distributions over velocities and height of retention with the velocity specified for uniform initial distribution ($\alpha = 0.5$): a) for absolute velocities: 1) $w_0 = \sqrt{x + w_x^2}/(1 - \alpha)$; 2) $w_0 = \sqrt{x + w_x^2}/(1 + \alpha)$; 3) $\varphi(w_0)$; 4, 5) $\varphi(w_x)$ for x = 0.5, 1; 6-8) $\varphi_w(x)$ for w_x = 0, 0.5, 1; b) for relative velocities: 1) $w_0 = \sqrt{x/(1 - v_x^2)}/(1 - \alpha)$; 2) $w_0 = \sqrt{x/(1 - v_x^2)}/(1 + \alpha)$; 3-5) $\varphi(v_x)$ for x = 0.1, 0.5, 0.9; 6-8) $\varphi_v(x)$ for v_x = 0, 0.5, 0.9.

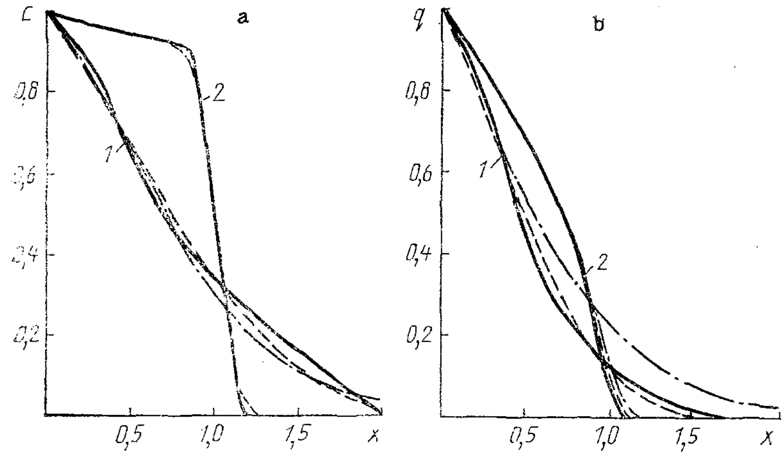

Fig. 2. Concentration (a) and particle flow (b) as functions of height within the confines of the dynamic ejecta zone: 1) $\alpha = 0.1$; 2) 0.75; solid curves) uniform distribution; dashed curves) normal distribution; dot-dash curves) gamma distribution over particle ejecta velocities.

the bed. If the particle concentration is sufficiently small and if we can neglect the possibility of particle collisions, all velocity and height distributions are uniquely defined by the initial distribution over expulsion velocities from the surface of the pseudofluidized bed $\varphi(w_0)$ in accordance with the formula for the probability function distribution of some random quantity [12]:

$$\varphi(v_x) = \varphi[w_0(x,\ v_x)]\ \frac{\partial w_0}{\partial v_x},\quad \varphi(w_x) = \varphi[w_0(x,\ w_x)]\ \frac{\partial w_0}{\partial w_x},$$
$$\varphi_v(x) = \varphi[w_0(x,\ v_x)]\ \frac{\partial w_0}{\partial x},\quad \varphi_w(x) = \varphi[w_0(x,\ w_x)]\ \frac{\partial w_0}{\partial x}. \tag{1}$$

If we examine how the volume occupied by the representative number of particles established for the lower boundary for the space above the bed during the process of motion, it is possible to demonstrate that the relative concentration of the disperse material at a height x in the zone of dynamic ejecta is determined by the relationship

$$c(x) = \frac{P_x}{1 + \sigma_x(\bar{v}_x)}. \tag{2}$$

The dimensionless flow of particles to the specified height defined by concentration and velocity can be expressed as follows:

$$q(x) = \left\langle \frac{w_0 v_x}{1+\sigma_x(v_x)} \right\rangle P_x. \tag{3}$$

Thus, using formulas (1)-(3), we can determine all of the dynamic characteristics of the disperse phase in the space above the bed, provided that we know the law of motion for a single particle. In the case of rather large particles, when the force of resistance is substantially smaller than the weight of the particles, the relationship between the instantaneous absolute or relative initial velocity and the height at which the particles remain is, respectively, of the form

$$w_0 = \sqrt{x + w_x^2}, \quad w_0 = \sqrt{\frac{x}{1 - v_x^2}}. \tag{4}$$

Since the distribution of the particles on the basis of the initial velocities may be any quantity whatsoever, it is essential that we choose criteria which allow us to compare the results for various initial probability functions. In the present study the distribution $\varphi(w_0)$ is constructed in a manner such that we always have $\overline{w}_0 = 1$. As the condition of comparability for these results at various $\varphi(w_0)$ we assume equality for the dispersions of the original distributions. For purposes of utilizing in this study the initial uniform, normal, and gamma distributions, this condition is expressed by the following parametric relationship:

$$\sigma = \frac{\alpha}{\sqrt{3}}, \quad \beta = \frac{3}{\alpha^2}, \quad \sigma = \frac{1}{\sqrt{\beta}}, \tag{5}$$

where $0 \le \alpha \le 1$; $0 \le \sigma \le 1/\sqrt{3}$; $3 \le \beta < \infty$.

The values of $\alpha = 0$, $\sigma = 0$, $\beta \to \infty$ correspond to the causative distribution of the initial velocities. With $\alpha = 1$, $\sigma = 1/\sqrt{3}$, $\beta = 3$ (the Maxwell distribution) the range of values for w_0 is amounts to [0, 2], [0, ∞), [0, ∞) for the uniform, normal, and gamma distributions. The corresponding ranges of expulsion height values for the particles are [0, 4], [0, ∞), [0, ∞) for all three forms of distribution.

Formulas (1)-(4) were used to calculate the distribution functions over the velocities and heights at which the particles remained in place, as well as the average values of the velocities, the concentrations, and the particle flows in the case of uniform, normal, and gamma distributions based on the original velocities. The relationships obtained thereby have been combined into three program packages for all of the initial distributions employed.

Figure 1 shows how the distribution functions with respect to absolute (Fig. 1a) and relative (Fig. 1b) velocities are transformed with increasing ascent, and also with respect to the height of particle retention with given values of x and α for uniform initial distribution with respect to absolute velocities. We should take note of the fact that for the relative velocities the initial distribution is always causative.

Figure 2 shows the concentration c and the particle velocity q as a function of height for the uniform, normal, and gamma initial distributions for various levels of dispersion. The values of c(x) and q(x) are rather close to each other for various initial distributions exhibiting identical dispersion. Without refining the true nature of $\varphi(w_0)$, this allows us subsequently to utilize the relationships derived in finite form for all of the characteristics of the dispersion phase in the space above the bed in the simplest case of the initial uniform distribution.

The break in the curves in Fig. 2, particularly noticeable in the case of low dispersion in the initial distribution, is a consequence of the reduction in the number of particles in the ascending flow. With maximum dispersion for the initial distribution the relationship between particle concentration and height coincides with the exponential relationship [1-5] for virtually the entire space above the bed, with the exception of the outermost sections.

Figure 3 shows the experimental values of the concentration [6] and average velocity of the particles [13, 14] at various heights in the zone of dynamic ejecta as compared to the theoretical values for the uniform initial distribution. As we can see from the figure, the coincidence of theoretical and experimental data over the entire range of heights above the pseudofluidized bed is rather good (an average error of ~8%).

Thus, the proposed relationships allow us to calculate all of the required dynamic characteristics of the dispersion phase in the space above the bed for a given particle expulsion velocity out of the pseudofluidized bed and for the dispersion of this quantity.

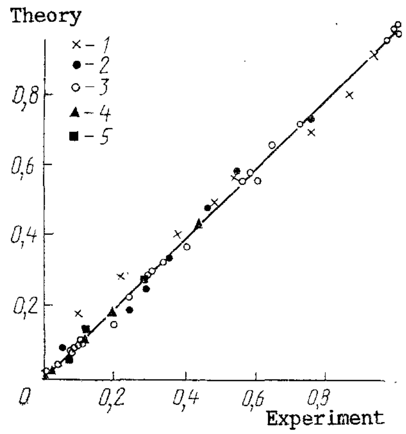

Fig. 3. Comparison of experimental data against the calculated results for concentrations (1-4) and average particle velocity (5): 1) data from [6]; 2-4) data from [13]; 5) data from [14].

NOTATION

C_0, concentration of particles expelled out of the bed; C_x, concentration of particles at height x; $c(x) = C_x/C_0$, dimensionless concentration of particles at height x; g, gravitational acceleration; P_x, probability of finding particles above height x; Q_0, flow of particles leaving the bed; Q_x, flow of particles at height x; $q(x) = Q_x/Q_0$, dimensionless flow of particles at height x; W_0, absolute particle expulsion velocity; W_x, absolute particle velocity at height x; $w_0 = W_0/\overline{W}_0$, dimensionless absolute particle expulsion velocity; $w_x = W_x/\overline{W}_0$, dimensionless absolute particle velocity at height x; $v_x = W_x(W_0)/W_0$, dimensionless relative particle velocity at height x; X, height of particle flow; $x = X/\sqrt{(\overline{W}_0)^2/2g}$, dimensionless height of particle flow; α, parameter of uniform distribution over flow velocities with dispersion $\alpha^2/3$; β, parameter of the gamma distribution over flow velocities with dispersion $1/\beta^2$; σ, parameter of normal distribution over flow velocities with dispersion σ^2; $\sigma_x(v_x)$, mean-square deviation for distribution over heights of particle retention, with relative velocity v_x; $\sigma_x(w_x)$, mean-square deviation for distribution over particle retention heights, with absolute velocity w_x; $\varphi(w_0)$, density of distribution over flow velocity; $\varphi(v_x)$, density of distribution over relative velocities at heights x; $\varphi(w_x)$, density of distribution over absolute velocities at height x; $\varphi_v(x)$, density of distribution over particle retention height, with relative velocity v_x; $\varphi_w(x)$, density of distribution over particle retention heights, with absolute velocity w_x; $\langle\rangle$, mean quantity.

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